A plausible model of recognition and postdiction in dynamic environment

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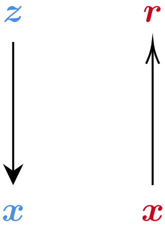
1. Introduction

Inference using an internal model (Helmholtz machine)

static world

$$p(\boldsymbol{z}, \boldsymbol{x}) \longrightarrow \boldsymbol{r}(\cdot)$$



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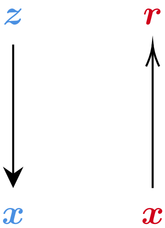

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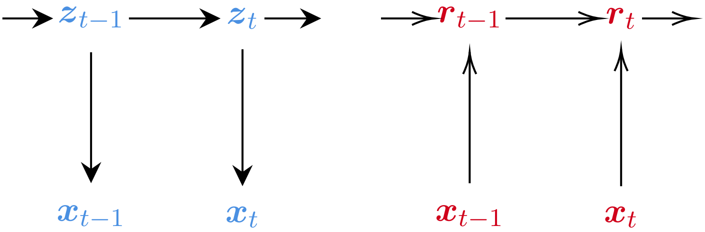

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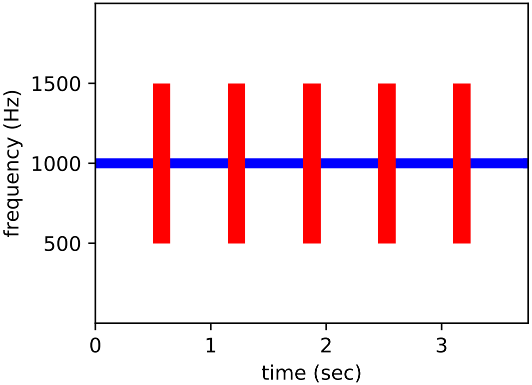

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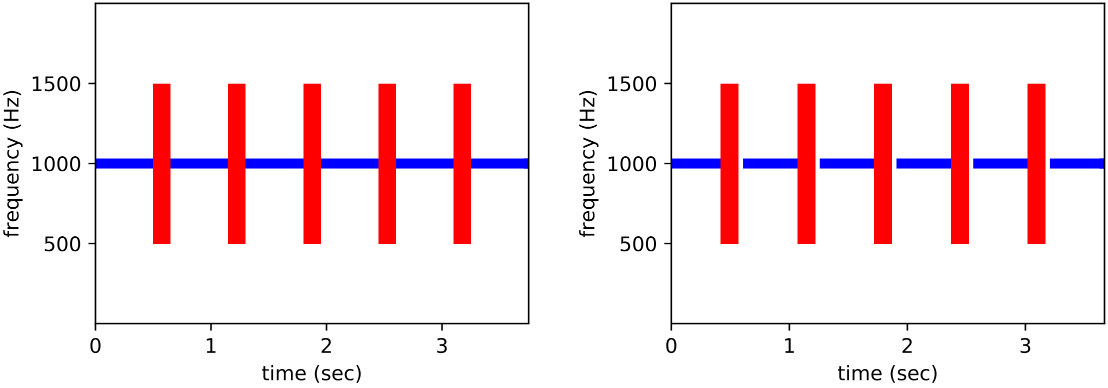
frequency (Hz)
1500
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Illusion 2: cutaneous rabbit

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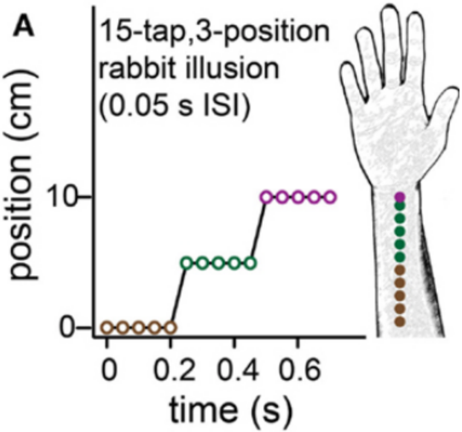

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- **learning** to do all the above

2. Distributed distributional code

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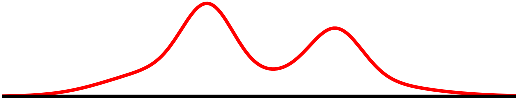
A DDC encodes a **probability distribution**:

$$q(\boldsymbol{z})$$



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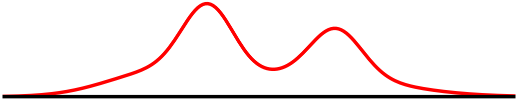

by a set of **tuning functions**

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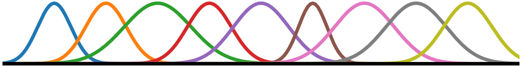

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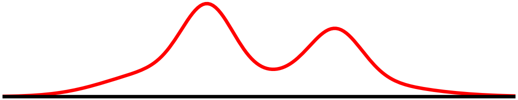

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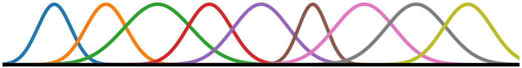

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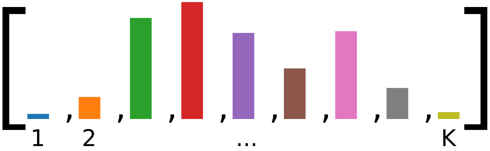

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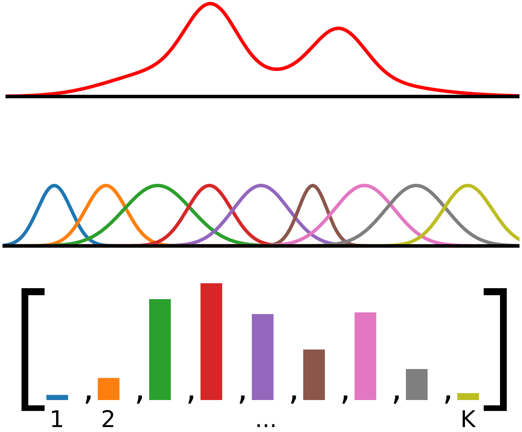

Zemel, Dayan & Pouget (1998); Sahani & Dayan (2003), Vértes & Sahani (2018)

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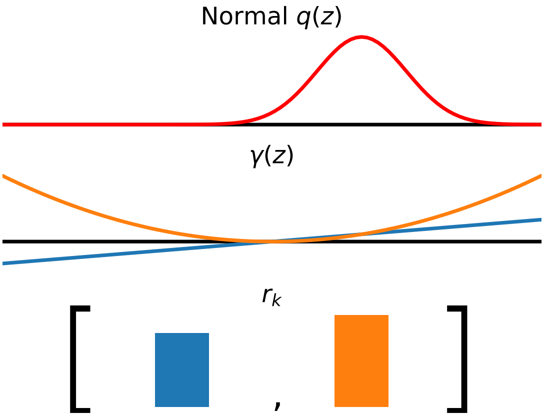

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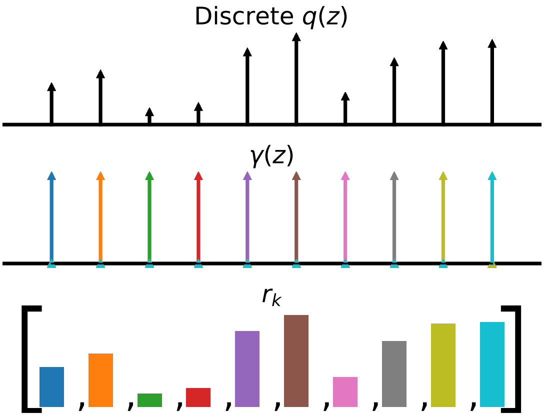

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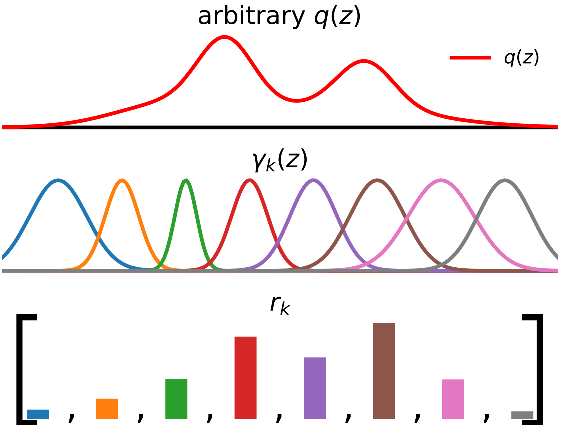

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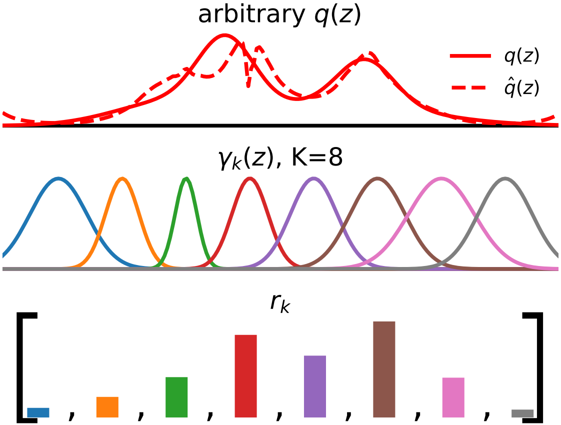

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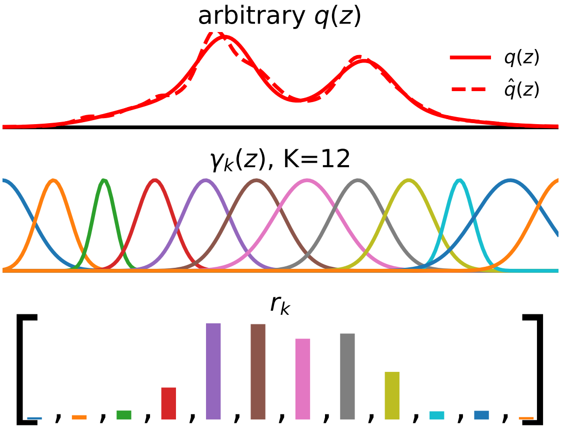

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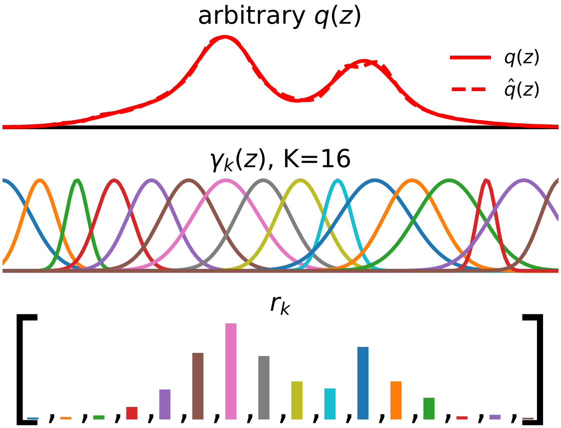

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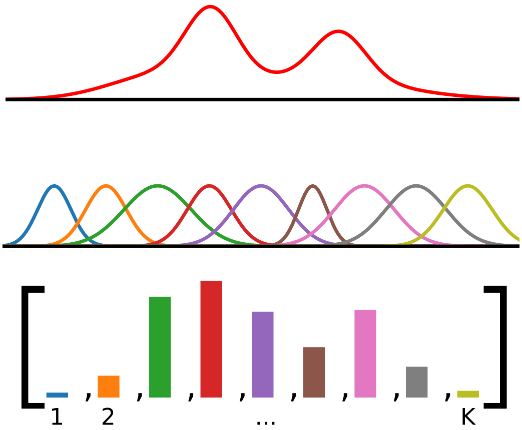

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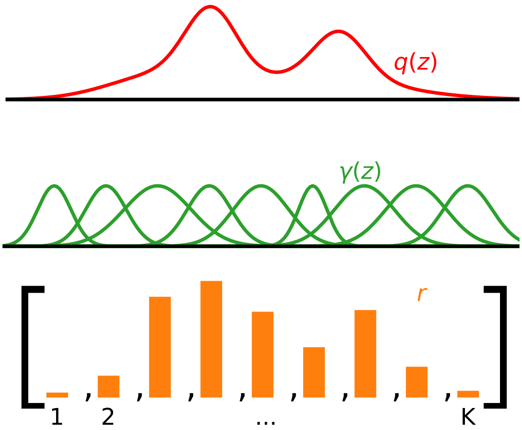

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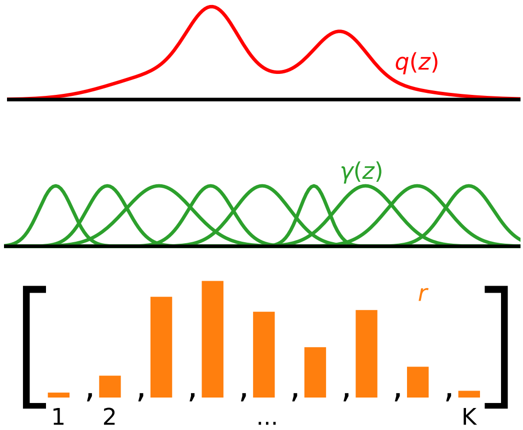

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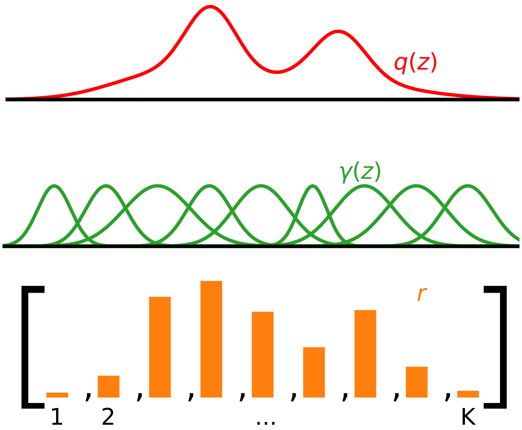

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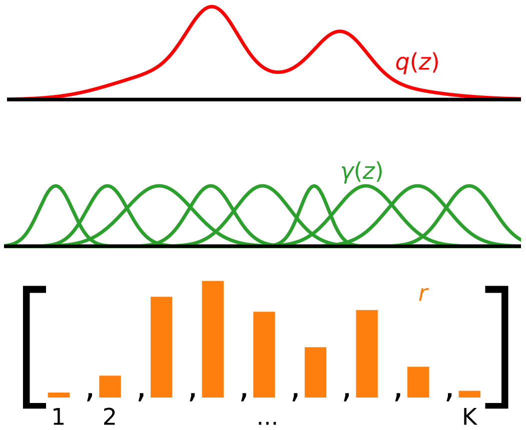

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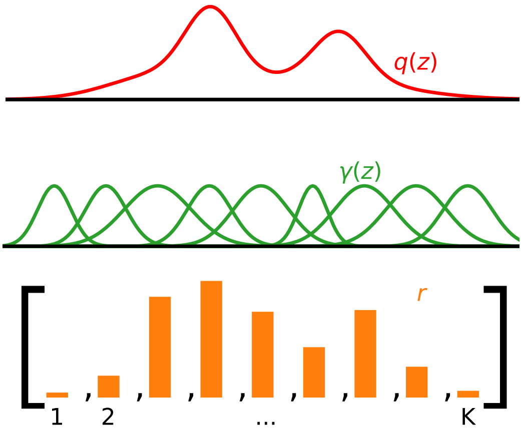

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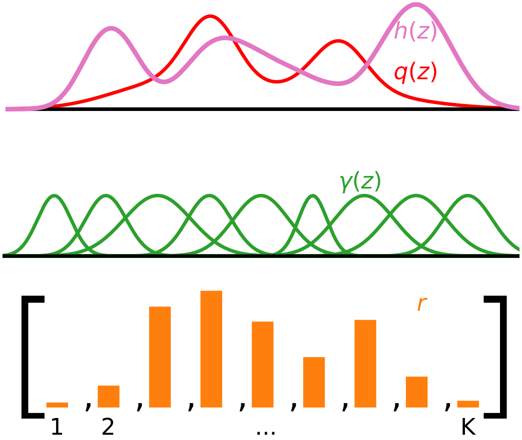

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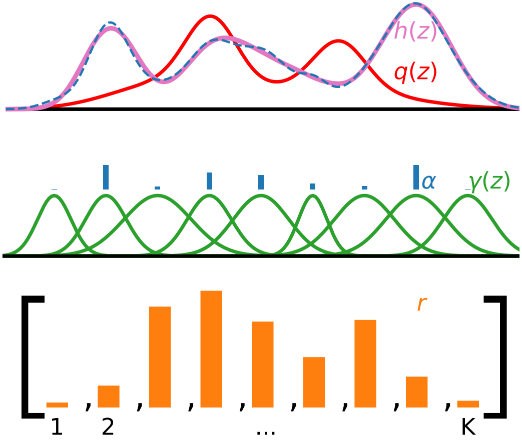

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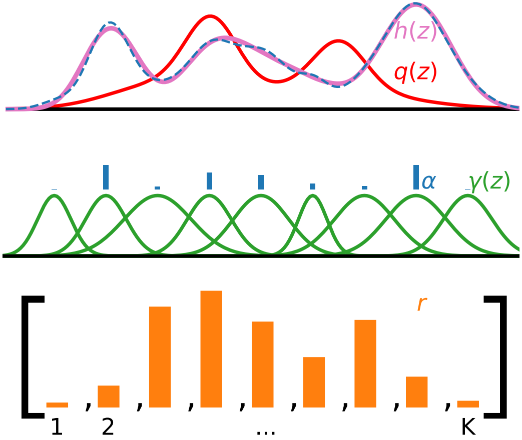

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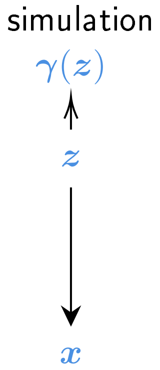

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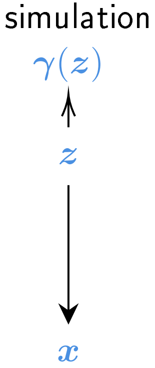

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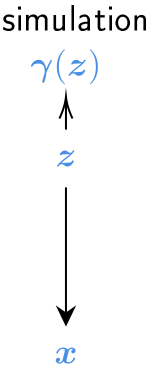

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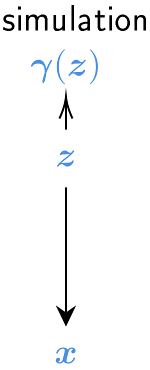

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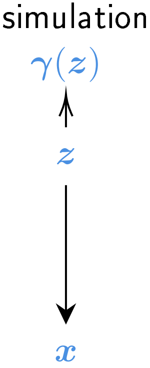

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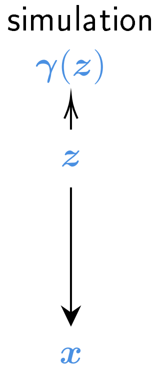

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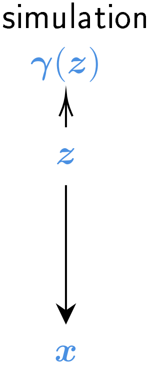

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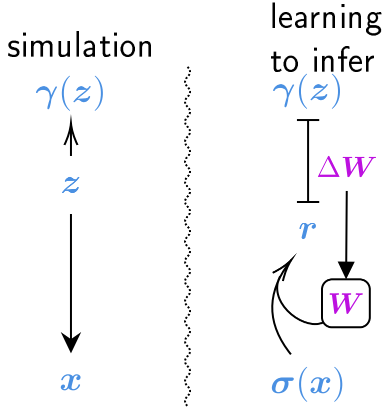

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Learning to infer given $p(z, x)$

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3. Online recognition and postdiction

A generic dynamic internal model

We assume a generic internal model

$$\boldsymbol{z}_t = \boldsymbol{f}(\boldsymbol{z}_{t-1}, \xi^{(z)})$$
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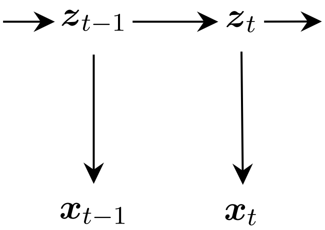

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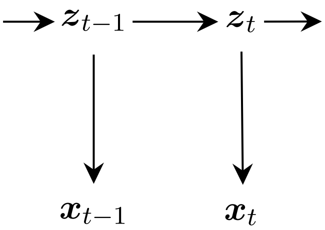

Assumptions

- Discrete-time
- Markov property
- Stationarity

Representing and computing beliefs of the whole history

- Online recognition (filtering): maintain $q(\boldsymbol{z}_t|\boldsymbol{x}_{1:t})$

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$$\boldsymbol{r}_t = \mathbb{E}_{q(\boldsymbol{z}_{1:t}|\boldsymbol{x}_{1:t})}\left[\boldsymbol{\psi}_t\right]$$

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- ***Postiction:*** readout statistics

$$h(\boldsymbol{z}_{t-\tau}) \approx \boldsymbol{\alpha} \cdot \boldsymbol{\psi}(\boldsymbol{z}_{1:t}) \implies \mathbb{E}_{q(\boldsymbol{z}_{t-\tau})}[h(\boldsymbol{z}_{t-\tau})] \approx \boldsymbol{\alpha} \cdot \boldsymbol{r}_t$$

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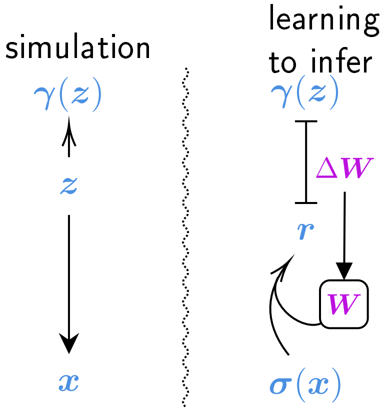

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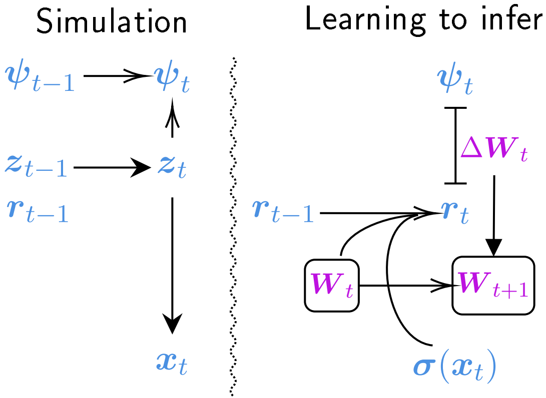

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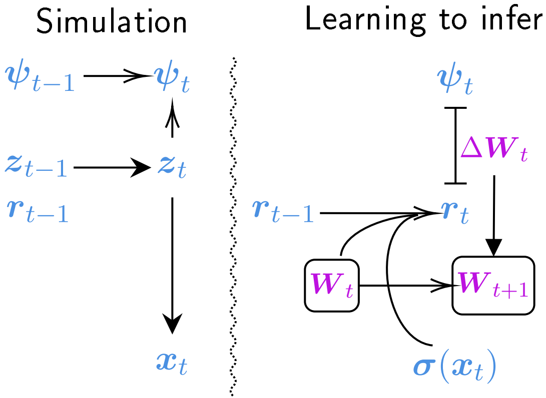

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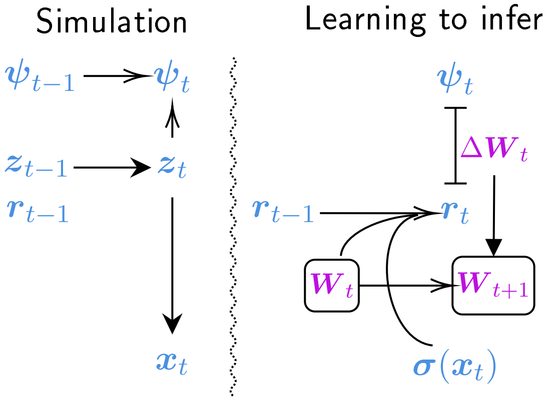

4. Testing DDC filtering on simulated experiments

Auditory continuity illusion

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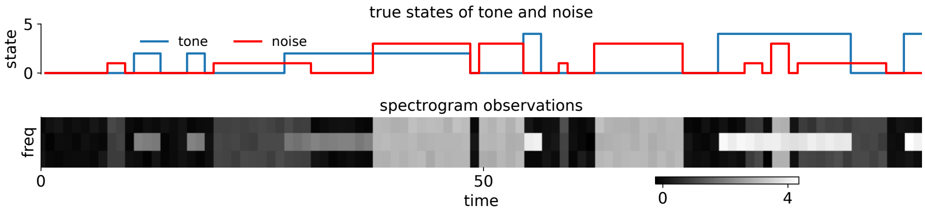

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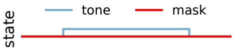

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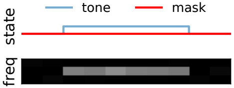

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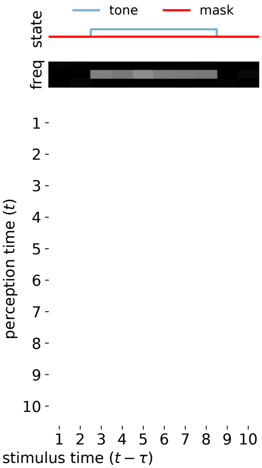

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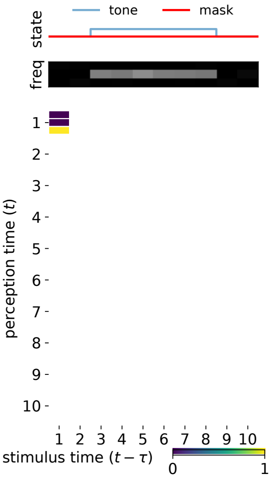

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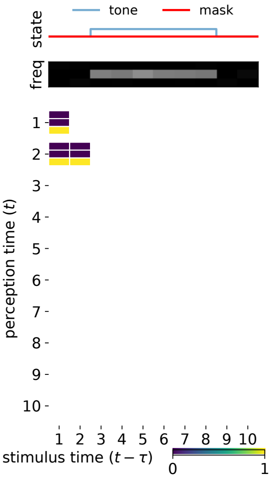

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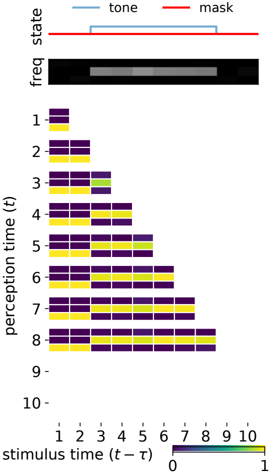

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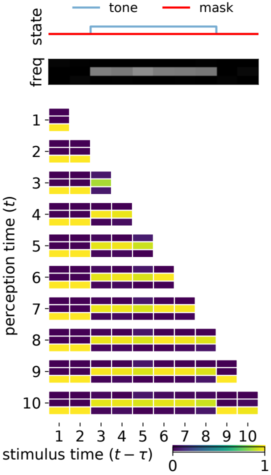

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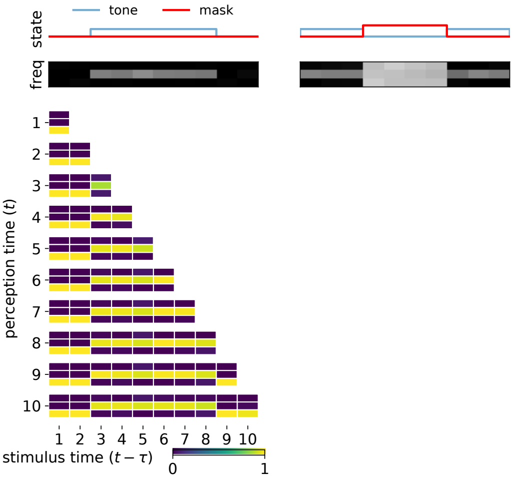

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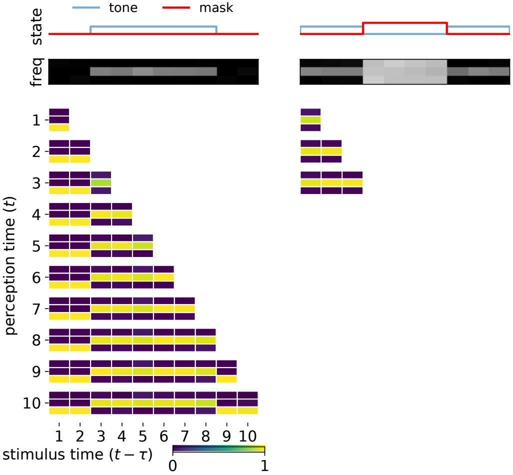

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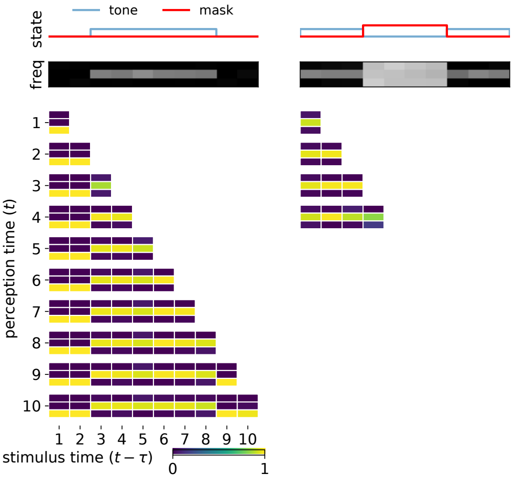

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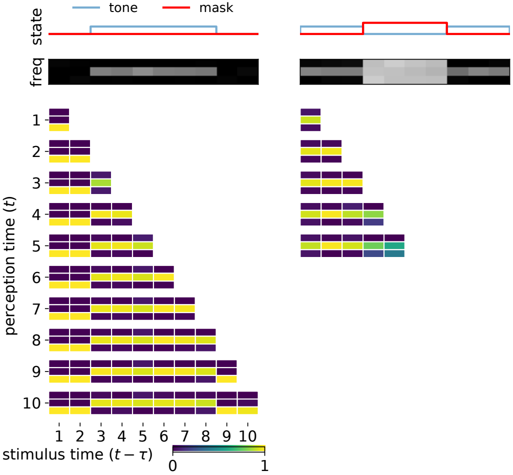

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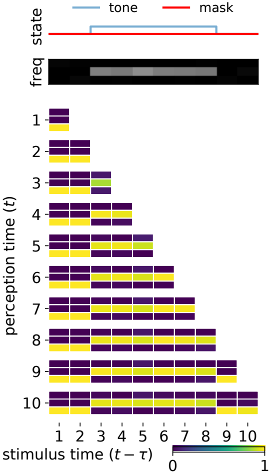

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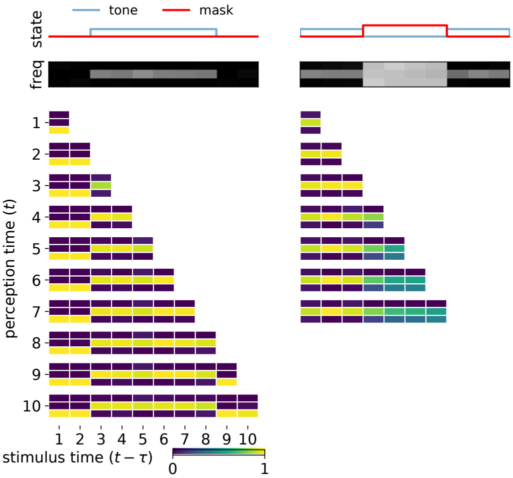

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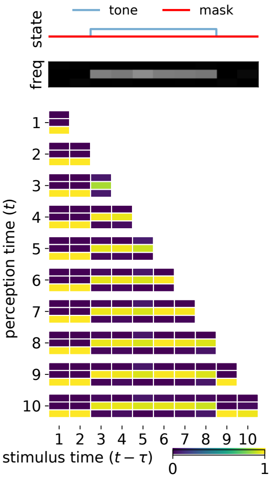

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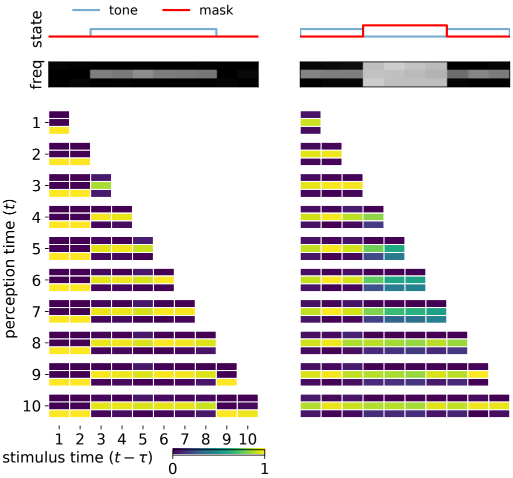

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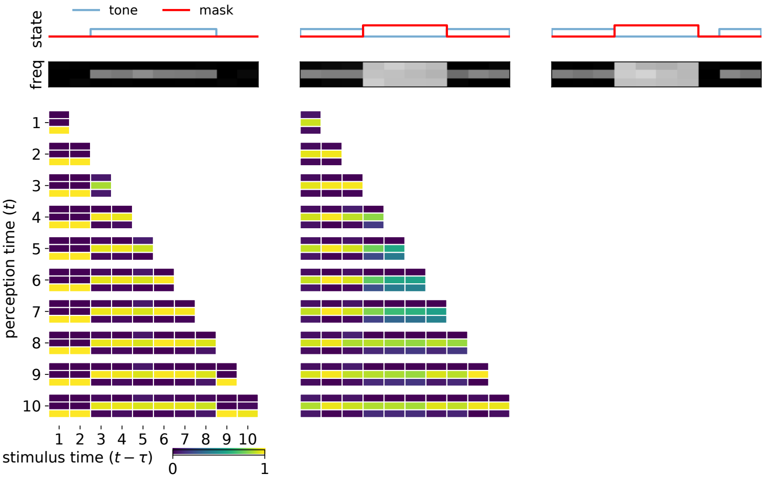

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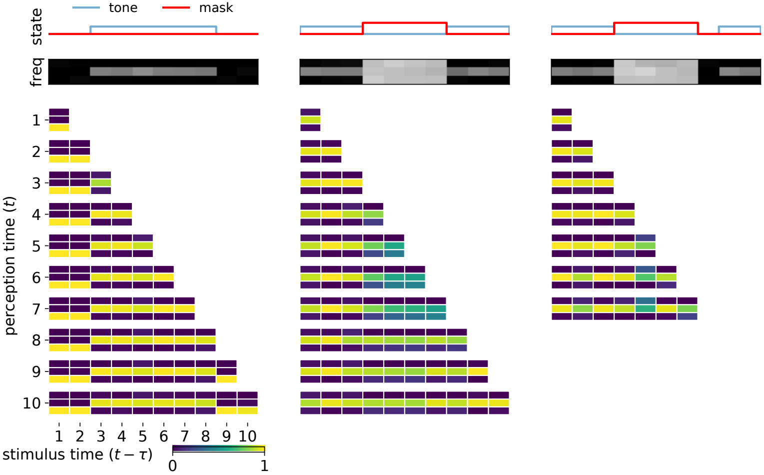

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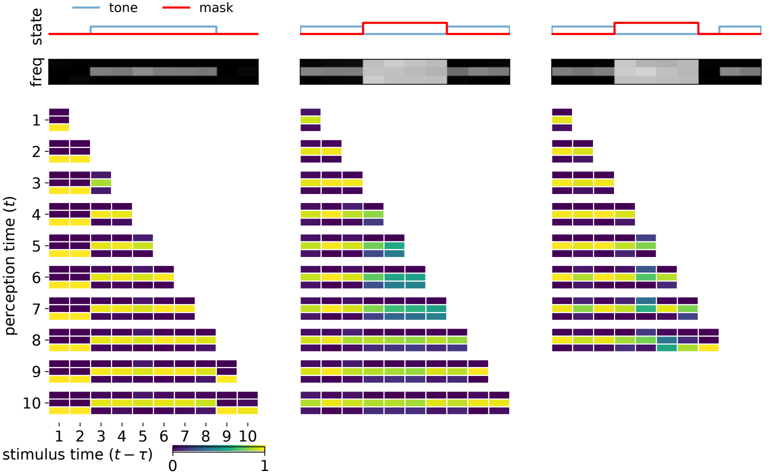

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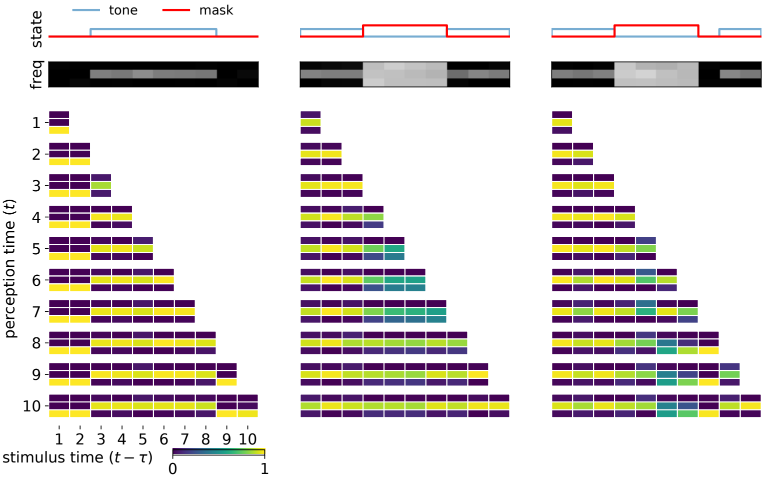

Cutaneous rabbit

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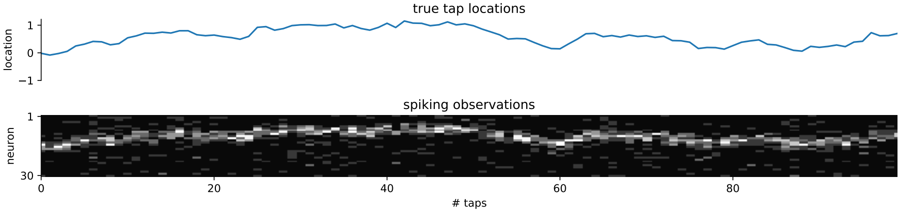

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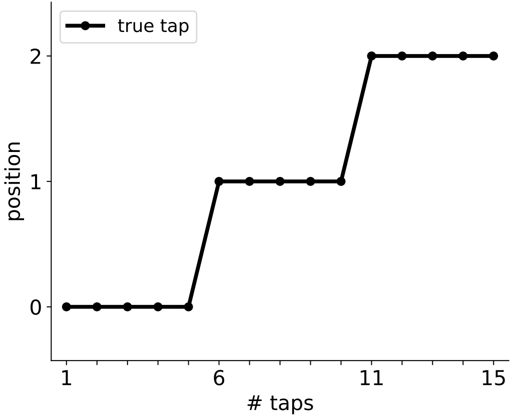

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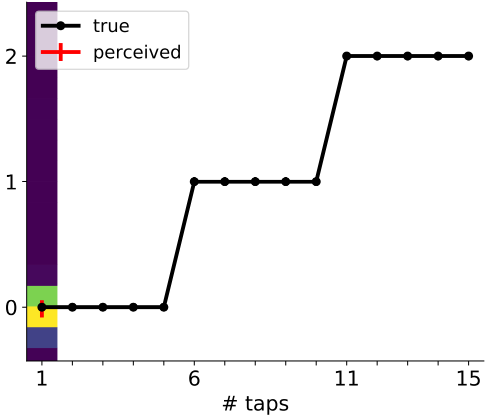

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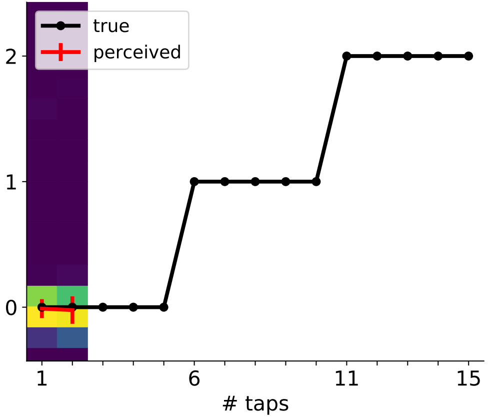

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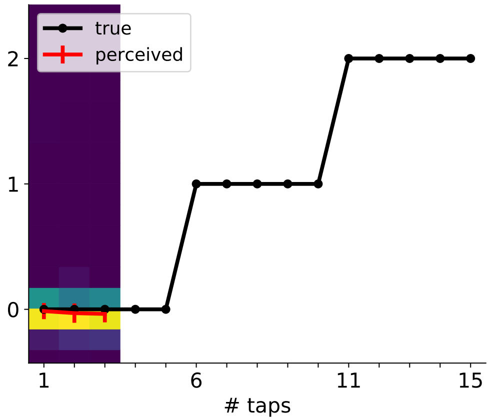

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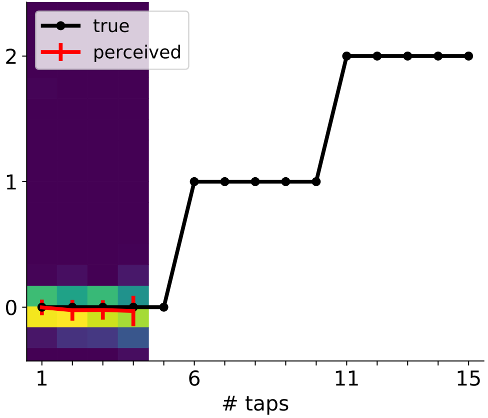

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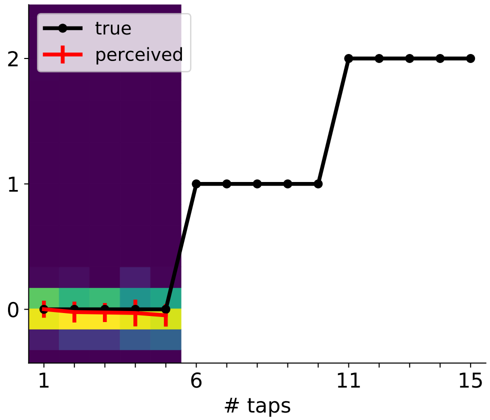

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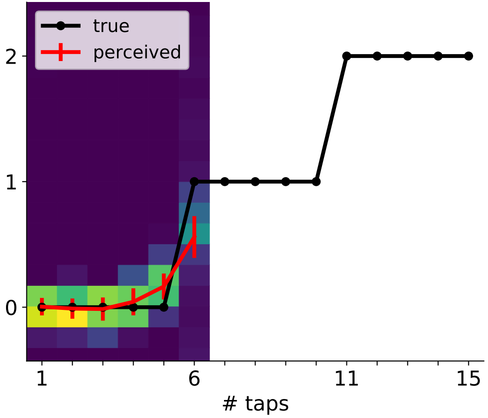

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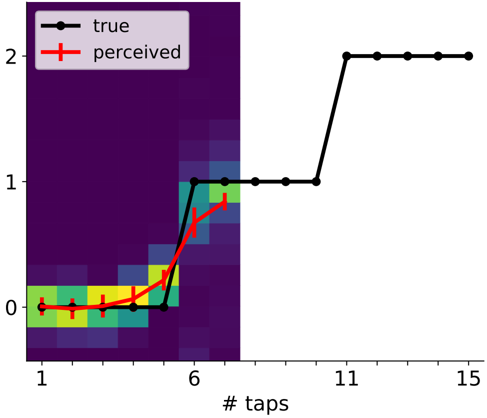

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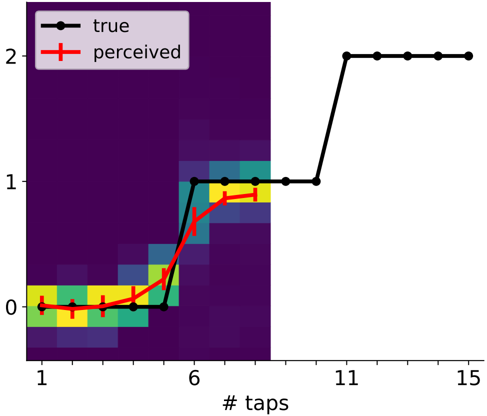

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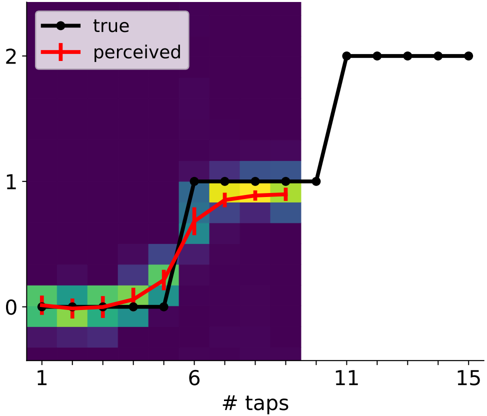

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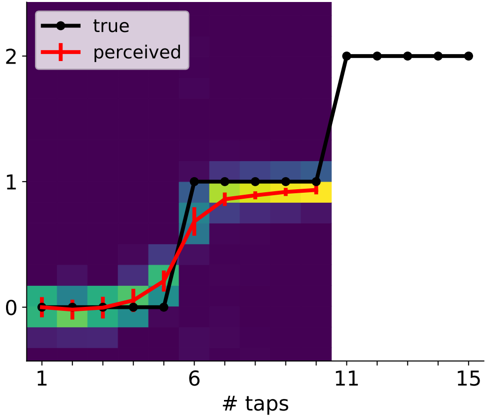

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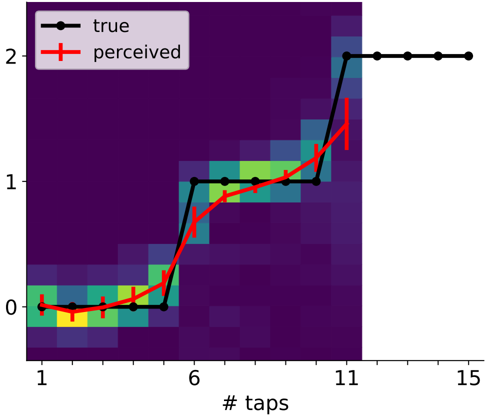

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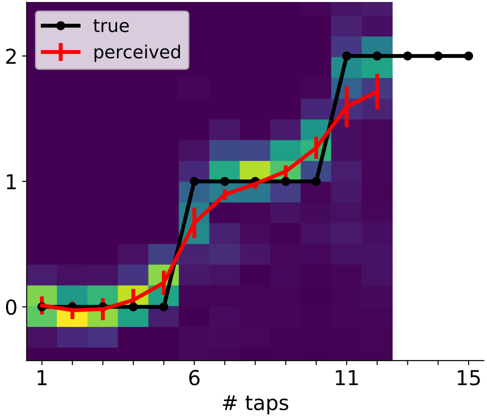

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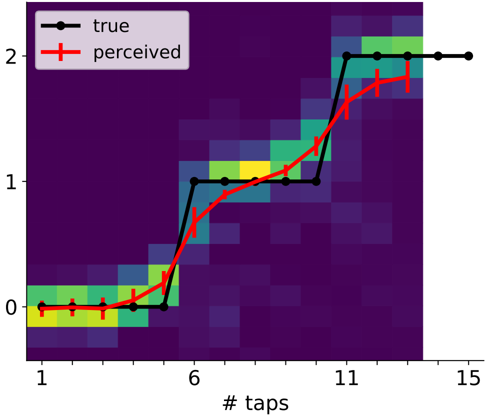

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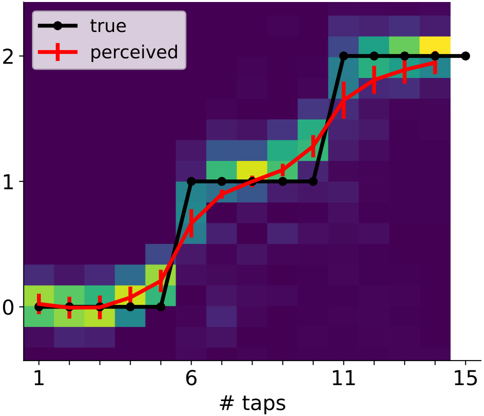

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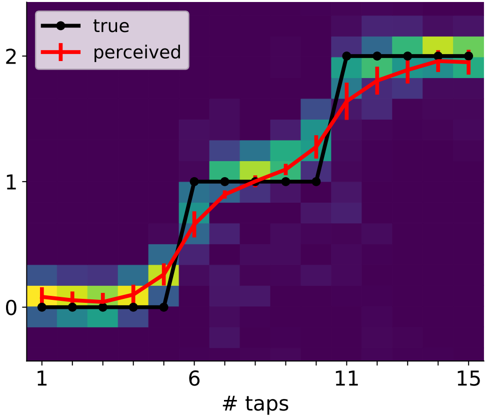

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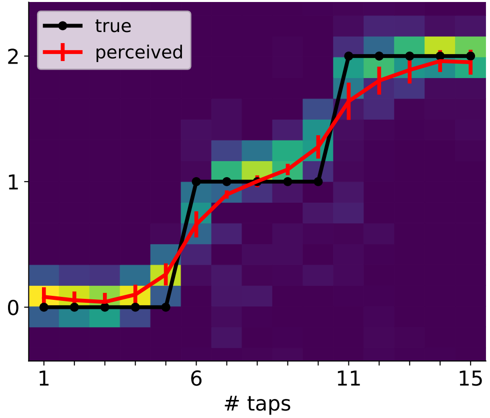

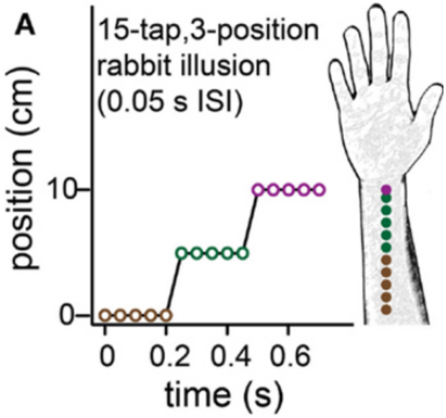

Summary of DDC filtering and Questions

Representation: DDC $\boldsymbol{r}_t := \mathbb{E}_{q(\boldsymbol{z}_{1:t}|\boldsymbol{x}_{1:t})}\left[\boldsymbol{\psi}_t(\boldsymbol{z}_{1:t})\right]$ **, or linear** $\mathbb{E}_q\left[h(\boldsymbol{z}_{t-\tau})\right] \approx \boldsymbol{\alpha}^{\mathsf{T}}\boldsymbol{r}_t$

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- Is there a way to also adapt $\boldsymbol{U}$ in a plausible way?
- Can we encode the internal model by DDC? (talk to Eszter Vértes)